1 Components of Series

Ex 1 Write out every term of $\sum_{n=1}^{5} \frac{1}{n^2+1}$

Ex 2 Write out every term of $\sum_{\ell=-1}^{4} \sqrt{\ell+1}$

Ex 3 Write out every term of $\sum_{k=0}^{3} \frac{1}{k!}x^k$

Ex 4 Write out every term of $\sum_{r=0}^{3} \frac{2^r}{3^r}x^{2r+1}$

Ex 5 Give the second term of $\sum_{n=1}^{9} \frac{2}{3n}$

Ex 6 Give the third partial sum of $\displaystyle\sum_{s=1}^{\infty} \frac{2^s}{s+1}$

Ex 7 Give the fifth partial sum of $\displaystyle\sum_{t=0}^{\infty} \frac{t^2-1}{t!}$

Ex 8 Give the fourth partial sum of $\displaystyle\sum_{u=1}^{\infty} \frac{1}{u^2 \cdot 2^u} \cdot x^u$

Ex 9 The series $\displaystyle\sum_{u=1}^{\infty} \frac{1}{u^2 \cdot 2^u} \cdot x^u$ is the Maclaurin series for some function $f(x)$. Give the fourth Taylor polynomial of $f(x)$ centered at $a = 0$.

Ex 10 The series $\displaystyle\sum_{u=1}^{\infty} \frac{1}{u^2 \cdot 2^u} \cdot x^u$ is the Maclaurin series for some function $f(x)$. Give the fourth term in the Maclaurin series of $f(x)$.

Ex 11 The series $\displaystyle\sum_{u=1}^{\infty} \frac{1}{u^2 \cdot 2^u} \cdot x^u$ is the Maclaurin series for some function $f(x)$. Give the fourth Taylor coefficient for $f(x)$ centered at $a = 0$.

Ex 12 Consider the series $\sum_{k=0}^{\infty} a_k$ where $a_k = \frac{1}{2^k}$

(a) Fill out the following table where a_k is defined above and S_k denotes the kth partial sum of the above series.

k	0	1	2	3	4	5	6	7	8
a_k									
S_k									

(b) What is $\lim_{k\to\infty} a_k$?

(c) What is $\lim_{k\to\infty} S_k$?

2 Convergence of Series

Ex 1 Does $\sum_{n=0}^{\infty} \frac{n}{n^3+1}$ converge or diverge?

Ex 2 Does $\sum_{k=0}^{\infty} \frac{k^3}{5^k}$ converge or diverge?

Ex 3 Does $\displaystyle\sum_{\ell=2}^{\infty} \frac{1}{\ell\sqrt{\ln(\ell)}}$ converge or diverge?

Ex 4 Does $\displaystyle\sum_{j=0}^{\infty} \frac{(-1)^j}{\sqrt{j+1}}$ converge or diverge?

Ex 5 Does $\displaystyle\sum_{a=1}^{\infty} \frac{\cos(3a)}{1+(1.2)^a}$ converge or diverge?

Ex 6 Does $\displaystyle\sum_{b=2}^{\infty} \ln\left(\frac{2b+1}{2b+3}\right)$ converge or diverge?

Ex 7 Does $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{3^n}$ converge or diverge...

(a) ...when $x = -1$?

(b) ...when $x = 2$?

(c) ...when $x = -3$?

(d) ...when $x = -6$?

(e) ...when $x = 1$?

(f) ...when $x = -5$?

(g) ...when $x = 0.99$?

(h) ...when $x = -4.99$?

Ex 8 Use your answers to the previous question to answer this question. What is the interval of convergence for $\sum_{n=0}^{\infty} \frac{(x+2)^n}{3^n}$? How about the radius of convervence?

Ex 9 How does the interval of convergence for $\sum_{n=0}^{\infty} \frac{(x+2)^n}{(n+1)\cdot 3^n}$ compare to the interval of convergence in the previous problem? How about the radius of convergence?

Ex 10 How does the interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{(n+1)^2 \cdot 3^n}$ compare to the interval of convergence in the previous problem? How about the radius of convergence?

3 Functions and Series

Ex 1 Let $f(x) = \frac{1}{1-x}$.

(a) Find a power series which converges to $f(x)$. (Yes, this is as easy as it looks.)

(b) Using the definition of the Maclaurin series, find the Maclaurin series for $f(x)$.

Ex 2 Find the Maclaurin series for each of the following functions. Also include *an interval on which that series converges*. You may use whatever methods you like, but you should practice using the geometric series at least once and using the definition of the Maclaurin series at least once.

(a) $\frac{x^2}{1+x}$

(b) $\arctan(x^3)$

(c) xe^{2x}

(d) 10^x

(e) $\frac{21}{(10-x)(1+2x)}$

Ex 3 Find an explicit formula (i.e. give a formula for the function that doesn't use a series) for each of the following series.

(a) $\sum_{n=0}^{\infty} x^n$

(b) $\sum_{n=1}^{\infty} x^n$

(c) $\sum_{n=0}^{\infty} \frac{3 \cdot x^n}{4^n}$

(d) $\sum_{k=0}^{\infty}(-1)^k x^{2k}$

(e) $\sum_{r=0}^{\infty} e^{rx}$

4 Remainder Estimates

Ex 1 Consider the series $\sum_{s=1}^{\infty} \frac{2s}{s+7}$. Using sigma notation, write out R_3. Write down the first four terms of R_3.

Ex 2 Consider the power series $\sum_{k=1}^{\infty} -\frac{x^k}{k}$. This is the Maclaurin series for $\ln(1-x)$.

(a) Write down the second Taylor polynomial for $\ln(1-x)$.

(b) Graph $\ln(1-x)$ and the second Taylor polynomial for $\ln(1-x)$ on the same axes.

(c) Write down (without using an infinite sum) the function $R_2(x)$.

(d) Write a sentence describing what $R_2(x)$ represents.

(e) Using a calculator, compute $R_2(1/2)$, $R_2(-1/2)$, $R_2(1/10)$, $R_2(-1/10)$, and $R_2(0)$. How does the size of $R_2(x)$ change as x approaches 0?

Ex 3 Consider the series $\sum_{n=0}^{\infty} \frac{3n^2}{1+n^6}$.

(a) Find k so that the kth partial sum of $\sum_{n=0}^{\infty} \frac{3n^2}{1+n^6}$ is within 0.00001 of the actual sum.

(b) Use your favorite computational software to give the corresponding approximation of the actual sum.

Ex 4 Consider the series $\sum_{\ell=0}^{\infty}(-1)^{\ell}\frac{1}{\sqrt{\ell^{3/2}+1}}$.

(a) Find k so that the kth partial sum of $\sum_{\ell=0}^{\infty}(-1)^{\ell}\frac{1}{\sqrt{\ell^{3/2}+1}}$ is within 0.00001 of the actual sum.

(b) Use your favorite computational software to give the corresponding approximation of the actual sum.

Ex 5 Consider the power series $\sum_{k=1}^{\infty} -\frac{x^k}{k}$. This is the Maclaurin series for $\ln(1-x)$.

(a) Use Taylor's estimate to find an upper bound for $|R_1(1/2)|$.

i. What are x, a, I, and k?

ii. What is $f''(z)$?

iii. Find an upper bound for $|f''(z)|$ for $z \in I$.

iv. Why is the previous part important?

(b) Use Taylor's estimate to find an upper bound for $|R_3(1/2)|$.

(c) Find a value of k so that $|R_k(1/2)| < 0.0001$.

(d) Find a value of k so that $|R_k(x)| < 0.0001$ for each $x \in [-1/2, 1/2]$.